

Algebra 215: Problem Sheet 1

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1. Dummit and Foote pg. 11 q 6, 7
2. Dummit and Foote pg. 22 q 15 (This completes the proof that one definition of $\langle S \rangle$ is a group.)
3. Dummit and Foote pg. 40 q 4, 17
4. Dummit and Foote pg. 49 q 15
5. Show that \mathbb{Q} is not cyclic.
6. Show that $\mathbb{Z} \times \mathbb{Z}$ is not cyclic.
7. Show that both definitions of $\langle S \rangle$ from class are equivalent.
8. Let $\sigma = (a_1 a_2 \dots a_k)$ be an elements of S_n . Let τ be another element of S_n . Prove that $\tau \sigma \tau^{-1} = (\tau(a_1) \tau(a_2) \dots \tau(a_k))$. Deduce that cycle decompositions are invariant under conjugation. Further, deduce that if ρ and ρ' are permutations with the same cycle decomposition, then there is an element τ such that $\rho' = \tau \rho \tau^{-1}$.