Problem Sheet 10

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1. Let the following be a commutative diagram. Assume that the rows in this diagram are exact sequences.

$$\begin{array}{ccccccccc} A & \xrightarrow{\phi} & B & \xrightarrow{\psi} & C & \xrightarrow{\eta} & D & \xrightarrow{\nu} & E \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle g} & & \downarrow{\scriptstyle h} & & \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle j} \\ A' & \xrightarrow{\phi'} & B' & \xrightarrow{\psi'} & C' & \xrightarrow{\eta'} & D' & \xrightarrow{\nu'} & E' \end{array}$$

   (a) Assume g, i are injective and f is surjective. Show that h is injective.

   (b) Assume g, i are surjective and j is injective. Show that h is surjective.

   (c) Deduce that if f, g, i, j are isomorphisms, then h is an isomorphism.

2. Show that a direct sum of modules is flat iff each summand is flat.

3. Show that free modules are flat.

4. Show that projective modules are flat.

5. Exercise 25 Pg. 406 Dummit and Foote

6. Show that the field of fractions of any integral domain is a flat module.

7. Show that $\mathbb{Q}$ is not a projective R-module

8. Show that a direct sum of modules is projective iff each summand is projective.