Sheet 1: functions and relations

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1. Show that a map $f\colon S \to T$ is injective if and only if there is a map $g\colon T \to S$ such that $g \circ f = id_S$.
2. Show that a map $f\colon S \to T$ is surjective if and only if there is a map $g\colon T \to S$ such that $f \circ g = id_T$.
3. Deduce that a map $f\colon S \to T$ is bijective if and only if there is a map $g\colon T \to S$ such that $g \circ f = id_S$ and $f \circ g = id_T$.
4. For each of the following sets S and relations R, show whether R is a) reflexive, b) symmetric, c) transitive.
   - $S = \mathbb{R}$ and xRy if and only if $x < y$.
   - $S = \mathbb{R}$ and xRy if and only if $x \leqslant y$.
   - $S = \mathbb{R}$ and xRy if and only if $x - y \in \mathbb{Z}$.
   - $S = \mathbb{Z}$ and xRy if and only if $x^2 = y^2$.
   - $S = \mathbb{Z}$ and xRy if and only if $x > 0$ and $y > 0$.
   - $S = \mathbb{R}^2$ and $(a, b)R(c, d)$ if and only if $ad = bc$.
   - $S = \mathbb{N}$ and xRy if and only if 2 divide x or 2 divides y.
5. For the above relations which are equivalence relations, find the equivalence classes.
6. Let S be a set and $\{X_i\}$ be a partition of S. Define a relation R on S by xRy if x and y are in the same part of the partition.

   Show that this is an equivalence relation.

   What are the equivalence classes of this relation?