Optional Sheet on Rings

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1. Show that the direct product of two rings is a ring. I.e. If R, S are rings, then $R \times S$ is a ring with addition $(r, s) + (r', s') = (r + r', s + s')$ and $(r, s)(r', s') = (rr', ss')$.
2. Show that if R, S are rings, then $R \times S$ has zero-divisors.
3. Show that $R = \{a + bi \mid a, b \in \mathbb{Z}\}$ is a subring of $\mathbb{C}$.
4. Show that $(\{a + bi \mid a, b \in \mathbb{Z}\}, +)$ is isomorphic to $\mathbb{Z} \times \mathbb{Z}$ as an abelian group.
5. Show that R is not isomorphic to $\mathbb{Z} \times \mathbb{Z}$ as a ring.
6. Show that the ideals $n\mathbb{Z}$ and $m\mathbb{Z}$ are coprime if and only if n, m are coprime in the usual sense.
7. Show that the prime ideals in $\mathbb{Z}$ are $p\mathbb{Z}$ where p is a prime.
8. Show that $I \times J$ is an ideal of $R \times S$ where I is an ideal of R and J is an ideal of S.
9. Use the isomorphism theorem to show that $(R \times S)/(I \times J) \cong R/I \times S/J$.