

Problem Sheet 1

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1. Draw the Cayley graphs for

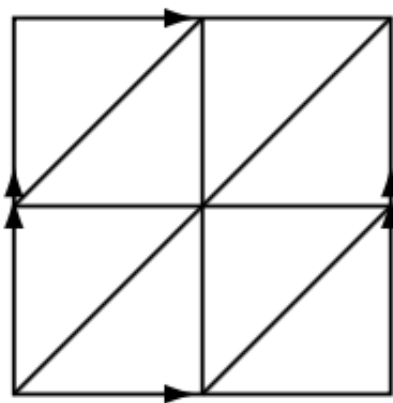
(a) $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ with generating set $S = \{(0, 1), (1, 0)\}$.

(b) $G = \mathbb{Z} \times \mathbb{Z}$ with generating set $S = \{(0, 1), (1, 0)\}$.

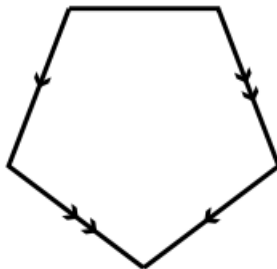
2. Prove that the triangulation $K = (\{1, 2, 3\}, \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}\})$ of S^1 is minimal.

Hint: First show that a simplicial complex with vertex set V embeds into the simplex of dimension $|V| - 1$.

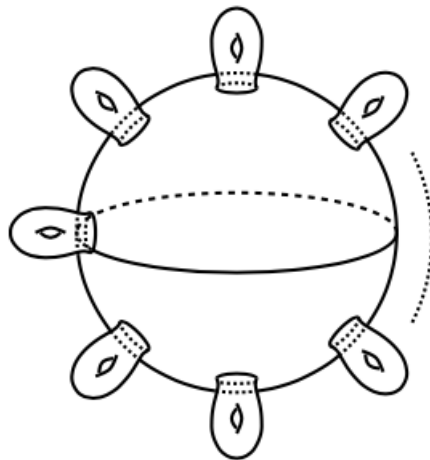
3. Why is the following diagram not a triangulation of $T^2 = S^1 \times S^1$?



4. (a) Show that a torus with a disk removed is homeomorphic to the image depicted below.



- (b) Deduce that the torus with a disk removed can be given a cell structure with 1 0-cell, 3 1-cells and 1 2-cell.
- (c) Let S be the surface of genus g depicted below.



Show that this can be constructed as follows. Start with a $4g$ -sided polygon, and identify its sides in pairs, according the following recipe:



[Hint: divide the polygon up into g pentagons and a g -sided polygon.]

- (d) Deduce that the surface of genus g can be given a cell structure with 1 0-cell, $2g$ 1-cells and 1 2-cell.
5. (Optional) Give a cell structure on the 3-torus $S^1 \times S^1 \times S^1$.