Algebraic Topology: Problem Sheet 2

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February 13, 2017

1. Let $\alpha : S^n \to S^n$ be the antipodal map ($\alpha(x) = -x$). Prove if n is odd, then $\alpha \simeq id$.

2. Let $f, g : X \to S^n$ be maps satisfying $f(x) + g(x) \neq 0$ for all $x \in X$. Prove $f \simeq g$.

3. Let X be a contractible space, Y be path connected. Prove each of the following:

   (a) X is path connected.

   (b) $X \times Y \simeq_{h.e.} Y$.

   (c) Any 2 maps $f, g : Y \to X$ are homotopic.

   (d) Any 2 maps $k, l : X \to Y$ are homotopic.

4. Show the following 3 spaces are homotopy equivalent.

   (a) The union of 2 circles with one point identified (A figure 8 graph).

   (b) A torus with a disk removed (cf. Problem sheet 1)

   (c) $\mathbb{R}^2 \smallsetminus \{(-1, 0), (1, 0)\}$.

5. Let $f, g : S^m \to S^n$ be two maps. Prove if $m < n$, then $f \simeq g$. (Hint: Use the simplicial approximation theorem.)

6. (Optional) Let $f, g : S^{n-1} \to X$ be maps such that $f \simeq g$. Prove $X \cup_f D^n \simeq_{h.e.} X \cup_g D^n$.