Algebraic Topology Problem Sheet 5

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1. Let $G_1 = \langle X_1 \mid R_1 \rangle$ and $G_2 = \langle X_2 \mid R_2 \rangle$. Give a presentation (with proof) for $G_1 \times G_2$.

2. Let G_0 be generated by a set S and let G_1, G_2 have presentations as above. Let $\phi_i \colon G_0 \to G_i$ be homomorphisms. Show that $G_1 *_{G_0} G_2$ has a presentation of the form $\langle X_1 \cup X_2 \mid R_1 \cup R_2 \cup \{\phi_1(s) = \phi_2(s) : \forall s \in S\} \rangle$.

3. Show the pushout of the following diagram is isomorphic to $\mathbb{Z}$.

$$\begin{array}{ccc} \mathbb{Z} & \xrightarrow{id} & \mathbb{Z} \\ \big\downarrow{\scriptstyle \times 2} & & \\ \mathbb{Z} & & \end{array}$$

4. Show that $\langle x, y \mid xyx = yxy \rangle$ is isomorphic to the pushout of the diagram below.

$$\begin{array}{ccc} \mathbb{Z} & \xrightarrow{\times 3} & \mathbb{Z} \\ \big\downarrow{\scriptstyle \times 2} & & \\ \mathbb{Z} & & \end{array}$$

5. Show that $G_1 = \langle a, b \mid aba = b \rangle$ is isomorphic to $G_2 = \langle c, d \mid c^2 d^2 \rangle$. Hint: in G_1 we have the equality $(abab)b^{-2} = e$

6. (Optional) Assume $\phi_1 : G_0 \to G_1$ is surjective. Show that $G_1 *_{G_0} G_2$ is a quotient of G_2.