

# Problem Sheet 6

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1. Let  $X = S^1 \vee S^1$ , and identify  $\pi_1(X, x)$  with the free group on the set  $\{a, b\}$ .

Construct the cover of  $X$  corresponding to the subgroup  $\langle a \rangle$ .

2. Construct the cover of  $X$  corresponding to the kernel of the following map:

$$\begin{aligned}\phi : F(a, b) &\rightarrow S_3 \\ a &\mapsto (12) \\ b &\mapsto (23)\end{aligned}$$

3. Let  $Y = S^1 \vee S^1 \vee S^1$ , and identify  $\pi_1(Y, y)$  with the free group on the set  $\{a, b, c\}$ .

Construct the cover of  $Y$  corresponding to the subgroup  $\langle a, b \rangle$ .

4. Construct a simply connected covering of  $S^1 \vee S^2$ .
5. Construct a simply connected covering of the Mobius band.
6. (Optional) Prove that there are 13 index 3 subgroups of  $F(a, b)$ , of which 4 are normal.

Hint: Use covers.