Problem Sheet 1

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1. Draw the Cayley graphs for

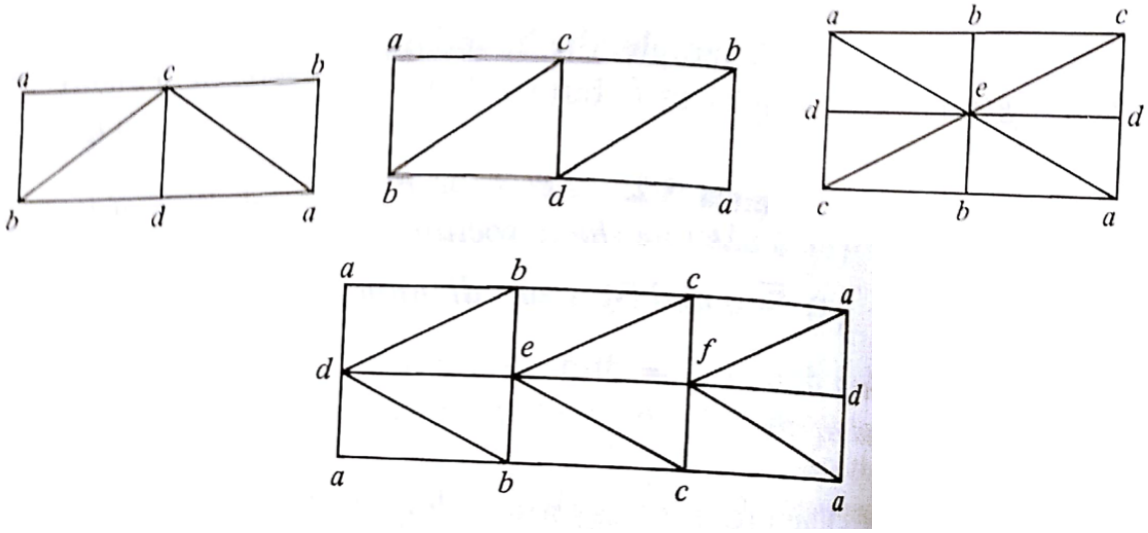
   (a) $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ with generating set $S = \{(0,1),(1,0)\}$.

   (b) $G = D_8$, the dihedral group of order 8 (symmetries of a square) with generating set $S = \{\sigma, \tau\}$, where σ is a rotation of order 4 and τ is a reflection.

2. Prove that the triangulation $K = (\{1,2,3\}, \{\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\},\emptyset\})$ of S^1 is minimal.

   Hint: First show that a simplicial complex with vertex set V embeds into the simplex of dimension $|V| - 1$.

3. Consider the following diagrams (all triangles should be considered filled in).



   (a) For each of the above figures justify whether it is a triangulation of a space.

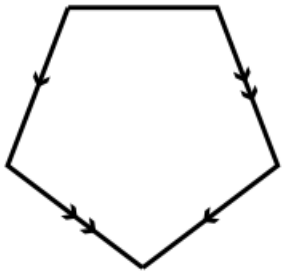
   (b) For the above figures which are simplicial complexes describe the triangulated space.

   (c) For the figures which are not triangulations consider the simplicial complex (V, Σ) where V is the vertex set and Σ consists of all subsets appearing in the figure. Describe the space obtained this way.

   All the spaces described above are easy to describe and should be spaces that you have met before/can draw.

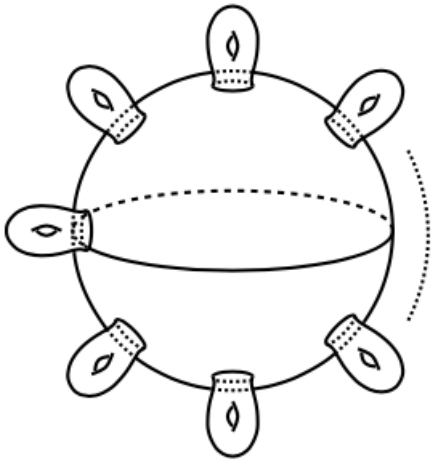
4. Let K be a simplicial complex with n simplices. What is the maximal possible dimension of K.

5. (a) Show that a torus with a disk removed is homeomorphic to the image depicted below.

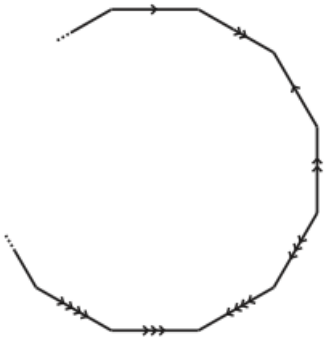


(b) Deduce that the torus with a disk removed can be given a cell structure with 1 0-cell, 3 1-cells and 1 2-cell.

(c) Let S be the surface of genus g depicted below.



Show that this can be constructed as follows. Start with a $4g$-sided polygon, and identify its sides in pairs, according the following recipe:



[Hint: divide the polygon up into g pentagons and a g-sided polygon.]

(d) Deduce that the surface of genus g can be given a cell structure with 1 0-cell, $2g$ 1-cells and 1 2-cell.

6. (Optional) Give a triangulation of the cell complex below. This complex is the known as the dunce hat.

7. (Optional) Give a cell structure on the 3-torus $S^1 \times S^1 \times S^1$.

8. (Optional) Given $v \geq 1, e \geq 0$ and $f \geq 1$ with $e \geq v$ such that $v - e + f = 2$ find a cell structure on S^2 with v vertices, e edges and f 2-cells

9. (Doubly Optional) Remove the assumption $e \geq v$ from the previous question.